**Homework 2 Solutions**

1. You must use the Binomial Option Pricing Model to solve this problem. If the price of the stock goes up to $45, the call option will be in the money and will be worth $15 (45 – 30). If the price of the stock goes down to $25, the call option will be out of the money and will be worth nothing. The swing of the call is 15 (15-0) and the swing of the stock is 20 (45-25) so the delta is .75. If you buy ¾ of a share of the stock, your payoff in one year will be $33.75 if the stock price goes up and $18.75 if the stock price goes down. Thus, you will need to borrow the present value of $18.75 for one year. 18.75/e^.04 = 18.0148 which we will round to $18.01. Since it costs $22.50 to buy ¾ of a stock that is priced at $30, and you receive $18.01 when you borrow the PV of $18.75, the net cost to you is $4.49 to ensure that you get a cash flow of $15 if the stock goes up and $0 if the stock goes down. Since this is the payoff of the call, it must be the price of the call. The call will be priced at $4.49.

Put-call parity says that the price of a put will equal the price of the call minus the price of the stock plus the PV of a risk-free bond maturing on the call’s expiration date with a face value equal to the call’s strike price. The put and the call must be on the same stock, with the same expiration date, and the same strike price.

The price of the call is $4.49. The price of the stock is $30. The present value of $30 to be paid one year from now when the annual risk-free continuously compounded interest rate is 4% is $28.82. Thus, the price of the call must be $3.31.

1. Buying the put: The payoff will be $45 if the price of GM is zero. For every dollar GM’s price is above zero, the payoff for the put declines by $1 (slope is -1.00) until the price of GM is $45 at which time the payoff for the put is zero no matter how high GM’s price is.

Writing the call: The call will be out of the money with a payoff of zero if GM’s price is $50 or less. If GM’s price is above $50, the call is in the money and (for the writer) has a payoff of -$1 for every dollar that GM’s price exceeds $50 (slope is -1.00).

Combining these two options together, gives you the positive payoff from the put if GM’s price is below $45, a zero payoff if GM’s price is between (and includes) $45 and $50, and a negative payoff with a slope of -1.00 for every dollar that GM’s price exceeds $50. Since you only get a positive payoff if GM’s price goes below $45 that must be your expectation. If the cost to purchase the put is equal to the price you receive from selling the call, there will be a net cost of zero to pursue this strategy. No matter what the prices of the options are, the income you receive from writing the call will offset the price you must pay to purchase the put.

1. Using the Black-Scholes model, you should insert the following values:

Stock price $80

Exercise price $70

Risk-free rate 2.00%

Time 0.5 years

Use Solver to find the standard deviation that causes the call to have a value of $15. The solution is 41%

1. Using the same spreadsheet, change t so that it is 1.0 years. Use Solver to find the standard deviation that causes the price of the put to be $7.80. The solution is 43%

We see that the implied volatility for this stock is 41% over the next 6 months, but 43% over the next year. Since the Black-Scholes model assumes constant volatility over time until the option expires, it appears that investors expect this stock to have less volatility over the next 6 months than they expect over the following 6 months. They expect 41% over the next 6 months and 45% over the following 6 months. The model then shows us that they expect an average volatility of 43% over the next year.

1. Suppose on November 17, you buy fifteen (15) December Copper Futures contracts, which have a 100-ounce contract size. Your purchase price was $20 per ounce. Complete the following “Marking-to-Market” table if the Maintenance margin is 75% of the initial 5% margin. Assume that you withdraw any funds eligible for withdrawal due to accrued gains.
   1. What are the initial and maintenance margin amounts?

**Initial = 15 x 100 x 20 x 0.05 = $1,500**

**Maintenance = 1500 x 0.75 = $1,125**

* 1. At the end of November 21st, what is the total of funds withdrawn due to gains, if any?

**$825 could be withdrawn on Nov. 21 (after depositing $750 on Nov. 19 due to a maintenance call).**

* 1. If you wanted to close out your position at the end of November 21st, what would you do?

**Take a short position in 15 identical Dec. Copper Futures Contracts.**

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| Day | Futures Price | Daily Gain/Loss | Cumulative Gain/Loss | Margin Account Balance | Deposits or Withdrawals? |
| Initial | $20.00 | N/A | N/A | $1,500 | N/A |
| Nov 18 | $19.90 | -150 | -150 | 1350 | No |
| Nov 19 | $19.50 | -600 | -750 | 750 | Yes, Add 750 |
| Nov 20 | $19.40 | -150 | -900 | 1,350 | No |
| Nov 21 | $20.05 | 975 | 75 | 2,325 | Yes, Withdraw 825 |