**Put-Call Parity**

**Simplified Explanation**

Look at the diagrams of purchasing both a share of stock and a put on that stock (strike price of $50). If you purchase them at the same time and combine the graphs, note what it looks like.

Since you own the stock, if its value goes way up, you reap the benefits.

But, if the value goes down, you don’t need to worry – you can always sell the stock for $50. So it has unlimited upside value, but a floor value of $50. This is a protected put.

We can get the same shape graph if we do the following:

* Buy a call (strike price of $50)
* Buy a zero-coupon bond with a face value of $50 that matures on the same day that the option expires. Of course, such a bond will be selling at the PV of $50.

On the exercise date, you receive $50 from the bond for sure. If the price of the stock is below $50 that is all you receive. If the price of the stock is above $50, you also benefit from exercising your call.

Since the cash payoffs from these two strategies are identical, the cost to implement them must also be identical. If not, arbitrage will push up the price of one and push down the price of the other.

The result is Put-Call Parity:

Price of stock + Price of put = Price of call + PV of exercise price

Note that the put and the call must have the same exercise date and the same strike price.

We can rearrange the above formula:

Price of stock = Price of call – Price of put + PV of exercise price

The right-hand-side allows you to purchase a synthetic stock by buying a call, selling (writing) a put, and purchasing a zero-coupon bond.

Another rearrangement of the formula:

Price of stock – Price of call = – Price of put + PV of exercise price

The left-hand side is called, selling a covered call. You give someone the option to purchase the stock from you – and you already own the stock.

The right-hand side will give you the same cash flow.

**Formal Proof**

For European Options the following relationship holds true:

**C + K\*exp(-rT) = S + P**

Conditions: Both the Call option and the Put option are on the same underlying stock, have the same maturity, T, and have the same strike price K. The interest rate is r (continuously compounded). The present value of the strike price over the maturity period T is K\*exp(-rT).

Proof: Consider two portfolios that you can invest in at time t=0:

Portfolio 1 consists of buying the call option and buying a T-Bill with a face value equal to K and maturing at T. At time t=0, we have to pay $C to buy the call and $K\*exp(-rT) to buy the T-Bill (the price of a T-Bill is PV of the face amount).

Portfolio 2 consists of buying a stock and buying a put option. At time t=0, we have to pay $S to buy the stock and $P to buy the put option.

 We can calculate the cash flows to the two portfolios at time t=T:

Portfolio 1 consists of a call option and a T-Bill with a face value equal to K. We can calculate the total payoff to the portfolio by adding up the payoffs to the individual components. At time t=T, the payoff to the call option is max(ST -K, 0) and the payoff on the T-Bill is the face value K. This implies that if the price of the stock is less than K, the payoff on the portfolio is equal to $K and if the price of the stock at time is higher than K, the payoff on the portfolio is equal to $ ST.

Portfolio 2 consists of a stock and a put option. Again, we can calculate the total payoff to the portfolio by adding up the payoffs to the individual components. At time t=T, the payoff on the stock is simply equal to its price in the market ST. The payoff on the put option is max (K- ST, 0). This implies that if the price of the stock is less than K, the payoff on the portfolio is equal to $K and if the price of the stock at time is higher than K, the payoff on the portfolio is equal to $ST.

We capture the cash flows in a table as given below. Note that a negative sign implies a cash outflow and a positive sign implies a cash inflow.

 **Initial**  | **Value at Expiration**

 | If ST < K | If ST > K **.**

 Portfolio 1 | |

 | |

 Buy Call - C | 0 | (ST – K)

 | + | +

 Invest in T-Bill - Kexp(-rT) | K | K

 Face value $K | |

 | ------- | -------

 Net -(C + K\*exp(-rT)) | K | ST

 | |  **.**

 Portfolio 2 | |

 | |

 Buy Stock - S | ST | ST

 | + | +

 Buy Put - P | K - ST | 0

 | -------- | ------

 Net -(S + P) | K | ST

No-Arbitrage argument:

Portfolio 1 and Portfolio 2 have the same identical final cash flows for all possible ST. Since the two Portfolios have the same final payoffs, they \***must\*** have the same initial investment requirement. Otherwise arbitrage opportunities will exist (we will see an example shortly). From the discussion above, the initial costs of Portfolio 1 is C + K(-rT) and the initial costs of Portfolio 2is S + P. Therefore, C + K(-rT) must equal S + P.

Implications of put-call parity:

1. The search for arbitrage profits. Put-Call parity implies a strict relationship between the price for a call option, the price for a put option, the price of the stock, and the price of a T-Bill. If actual traded prices are such that put-call parity does not hold, it gives rise to an arbitrage trading strategy.

Consider for example the following prices/data:

C=4, P=2, S=50, K=48, r=5%, T=0.5. Note that Kexp(-r\*T)=46.81..

Calculating the cost of these portfolios, we find that:

S+P = $52 and C+Kexp(-rT)= 4+46.81=$50.81.

There is a discrepancy here. An arbitrage strategy is available that involves selling the relatively expensive Portfolio and buying the relatively cheaper Portfolio.

Cash flows;

 **Initial**  | **Value at Expiration**

 | If ST < K |If ST > K **.**

 Buy Cheap Portfolio | |

 | |

 Buy Call - 4 | 0 | (ST – K)

 Buy T-Bill - 46.81 | K | K

 Face value $K | |

 | |

Sell Expensive Portfolio | |

 | |

Short Stock + 50 | -ST | -ST Sell Put + 2 | -(K - ST) | 0

 ---------- | -------- | ------

 Net + 1.19 | 0 | 0

 The strategy nets you $1.19 right away at time t=0 with no cash flow obligations in the future. Free money! Investors would continue to do this trade till the prices adjusted and reach equilibrium.