# **Deriving Interest Rate Swaps**

Typically, the floating rate is set equal to the reference rate (3-month LIBOR for example) and the fixed-rate needs to be determined from there.

At the start of the swap, the present value of the fixed rate payments (which are know with certainty) are set equal to the present value of the expected floating rate payments (which are not known with certainty). So at the start of the swap, this is an even (expected) exchange of payments.

Of course, during the life of the swap, the actual (realized) floating rate payments will differ from those which were expected at the start of the swap. If the realized floating rates are higher than what was expected, the swap is profitable for the fixed-rate payer and unprofitable for the fixed rate receiver.

If the realized floating rates are lower than what was expected, the swap is profitable for the fixed-rate receiver and unprofitable for the fixed-rate payer.

This makes the swap a zero-sum transaction. Whatever one party gains, the other party loses the same amount.

A swap is actually a series of forward contracts where the fixed rates are averaged (geometric averaging) together to come up with one rate (the swap rate) for the entire term of the swap.

Thus, swap rates must not differ significantly from forward (or futures) contracts for the same reference rate.

Since the most-often used reference rate is 3-month LIBOR, the price of the Eurodollar Futures Contract is intricately tied to the swap rate.

Eurodollar Futures Contracts – How the Swap Rate is Derived

Traded on Chicago Mercantile Exchange (CME) with face value of $1 million

Contract settles to the 90-day LIBOR which is the yield on a 90-day Eurodollar time deposit.

For Eurodollar Time deposits, principal and interest are paid at maturity (not done on a discount basis).

Interest is based on actual days (not 30 days/month) but on a 360-day year.

For example: 5.25% for 45 days is .0525 (45/360) = .0065625 = .65625%

Eurodollar Futures settle by cash (of course, since you can’t deliver an interest rate) to LIBOR on the 2nd business day before the 3rd Wed. of the maturity month.

The price converges to 100 (1 – LIBOR) with a resolution of 1/4 of a basis point.

With $1 million per contract, each bp movement is worth $25.

**Example**:

95.00 means interest of 5% for 90 days = .0500 (90/360) = .0125

$1 million (.0125) = $12,500 interest due (you are locking-in a payment of $12,500 in exchange for $1million \* 90-day LIBOR)

94.99 means interest of 5.01% for 90 days = .0501 (90/360) = .012525

$1 million (.012525) = $12,525 interest due (note the difference of $25)

The implied interest rate is 100 minus the futures price

Since the Eurodollar Futures Contract converges to 90-day LIBOR, this allows you to lock-in a 90-day LIBOR rate with the futures contract.

Thus you can swap a variable-rate loan for a fixed-rate loan. (though the rate is fixed, the payments will vary slightly based on the actual number of days in the three-month period).

If your liability is indexed off of 90-day LIBOR, you can go short on a series of Eurodollar Futures Contracts (or go long if you have an asset indexed to 90-day LIBOR).

**Example**:

Suppose you owe LIBOR on a loan. This is a liability. Interest payments are to be made every 3 months for 9 months. The principal is $1 million.

You can short the Eurodollar futures contracts at 95.46, 95.57, and 95.62.

This means you will receive LIBOR and pay Fixed

This effectively locks in annualized rates of 4.54%, 4.43%, and 4.38%.

Suppose LIBOR turns out to be 4.7%, 4.8%, and 4.9% for those months.

Since the futures price converges to LIBOR, you will receive

95.46 – 95.3 = 0.16. That’s 16 bp at $25 per bp for a total of $400.

95.57 – 95.2 = 0.37. That’s 37 bp at $25 per bp for a total of $925.

95.62 – 95.1 = 0.52. That’s 52 bp at $25 per bp for a total of $1,300.

These cash payments offset the extra interest you must pay on the LIBOR rates (since LIBOR ended up being higher than what you locked-in to the futures contracts at).

For the first payment for example, you owe $1 million [.047 (90/360)] = $11,750

However, you received $400 from your futures hedge, so your net interest was $11,350

This is an effective rate of 4.54% because $1 million [.0454 (90/360)] = $11,350

You locked-in 4.54%, 4.43%, and 4.38% for the 3 three-month periods. We can use what we know about implied forward interest rates to find the one three-period rate (in this example, we will assume each three-month period is exactly 90 days), which is equivalent.

(1+ swap rate)3 = (1+.0454(90/360)) (1+.0443(90/360)) (1+.0438(90/360))

 = (1.01135) (1.011075) (1.01095)

 = 1.033747631

 Swap rate = .011125 which, when annualized is 4.45%.

So, 4.45% is now substitutable for the three 90-day rates. Note that it is the geometric mean of those three 90-day rates.

We have thus synthesized a swappable fixed-rate loan for our floating-rate loan.

This means that a 4.45% fixed-rate loan can be fairly swapped for LIBOR.

Calculating the Floating-Rate Payments for a Swap

The first floating rate payment for a swap is the only one known with certainty when the swap is initiated because the payment at the end of the first three months is determined by the reference rate (usually 3-month LIBOR) at the start of the first three months.

Suppose a three-year $100 million swap begins on Jan. 1, 2009. If, on 1/1/09, 3-month LIBOR is 4.05%, on 4/1/09, the floating-rate payment due is:

$100 million \* (4.05%) \* 90/360 = $1,012,500

Note that there are 90 days from 1/1/09 through 3/31/09.

This was the only floating-rate payment known with certainty on 1/1/09.

The next floating-rate payment, due on 7/1/09 will be based on the 3-month LIBOR rate on 4/1/09 and will be:

$100 million \* (4/1/09 3-month LIBOR) \* 91/360 = ? (note there are 91 days in this 3-month period.

To calculate the expected 3-month LIBOR as of 1/1/09, we look to the Eurodollar futures contract that settles on 4/1/09.

Suppose the contract is trading at 95.85. This means the Eurodollar futures rate is 4.15% (100 – 95.85 = 4.15). This means that an investor could, on 1/1/09, lock-in a 3-month forward rate to take effect three months from then of 4.15%. This will be our expected rate for that 3-month period. So our expected floating-rate payment is:

$100 million \* (4.15%) \* 91/360 = $1,049,028

To calculate the expected payment on 10/1/09, we look to the Eurodollar futures contract that settles on 7/1/09. Suppose that on 1/1/09 it is trading at 95.45. This means the expected rate (the forward rate) is 4.55% and the expected payment is

$100 million \* (4.55%) \* 92/360 = $1,162,778

We do this for each of the 12 payments over the three-year life of the swap.

Once we have the expected payments for the floating-rate, we need to calculate their present values. The zero-coupon discount rates we use are the 3-month rates we came up with from the Eurodollar futures contracts on 1/1/09 (except the first one, which was known with certainty on 1/1/09).

For example, the fixed-rate receiver is expected to have to pay $1,162,778 on 10/1/09 which is 3 three-month periods from 1/1/09, the day we are trying to calculate the swap rate.

PV = $1,162,778 . = $1,126,079

 (1 + .0405(90/360)) (1 + .0415(91/360)) (1 + .0455(92/360))

$1,126,079 is the present value of $1,162,778 when discounted at these three Eurodollar Rates.

Once we have all 12 expected payments determined, and have calculated their individual present values, we sum up their present values which is the (expected) value of the floating-rate payments of the swap. $14,052,917

The fixed-rate payments must have this same present value on 1/1/09 when they are discounted back at the same zero-coupon three-month rates that we found in the Eurodollar futures contracts. The fixed rate which will discount those payments back to 1/1/09 and have them equal to this present value is the **swap rate** that we are looking for.

Note that because there are an unequal number of days in each 3-month period, a fixed rate does not mean fixed payments. Still, we have all the information we need to solve for the swap rate. We can do this with solver. Set the PV of the fixed payments to be equal to the PV of the expected floating payments by changing the swap rate.

If we use the example on the spreadsheet, the swap rate is calculated as 4.9875%.

This will be the fixed rate for the swap with the variable rate being 3-month LIBOR

Note that because interest rates are expected to rise as of 1/1/09, the fixed rate is greater than the floating rate for the first three months. So when the swap is initiated, the fixed-rate payer knows that on 4/1/09, they will be paying to the fixed-rate receiver:

$100 million \* (4.9875% - 4.05%) \* 90/360 = $234,368

After that, the actual 3-month LIBOR rate will be substituted into the equation above as well as the actual number of days in the three-month period. If the result is positive, the fixed-rate payer will send a check to the fixed-rate receiver for that amount. If the result is negative, the fixed-rate receiver will send a check for the absolute value of the amount to the fixed-rate payer.

The value of a swap (to one of the parties) will always be the difference between the present value of the payments that party is receiving and the present value of the payments that party will be paying. The expected floating-rate payments and the discount rates to be used will be based on the Eurodollar futures contracts at the time of the valuation.

This means that if the Eurodollar futures contracts go up for any period, the expected interest rate has gone down and the new expected floating-rate payment is less than it previously was expected to be. Thus, the present value of the floating rate payments goes down relative to the present value of the fixed rate payments (whose payments don’t change, but their discount rates do). This gives the swap a positive value for the fixed-rate receiver and a negative (and equal) value for the fixed-rate payer. We can see this on the second worksheet of the example if, on 4/1/10, LIBOR is 4.00% and the Eurodollar Futures prices for the remaining months of the swap are all at 96.00. On that date, the value of the swap to the fixed-rate receiver will be $1,686,292. The value of the swap to the fixed-rate payer will be -$1,686,292.