**The Two-State (Binomial) Option Pricing Model**

Example: A stock is currently priced at $50. Its price a year from now will either be $60 or $40. The risk-free interest rate is 10 percent. What is the value of a call option with a strike price of $50?

We want to come up with two strategies which have identical cash flows. If they do, they must be priced the same.

First Strategy – Buy the call option.

If the stock has a value of $60, your cash flow is $10.

If the stock has a value of $40, your cash flow is $0.

Second Strategy – Buy ½ share of the stock and borrow the present value of $20 ($18.10)

 If the stock has a value of $60, your cash flow is $10

$30 from the half-share of stock minus $20 to repay the loan

 If the stock has a value of $40, your cash flow is $0

 $20 from the half-share of the stock minus $20 to repay the loan

Since the cash flows are identical, the cost of the first strategy must equal the cost of the second.

The cost of the second strategy is $6.90.

Cost of buying ½ share of stock = $25.00

Proceeds from borrowing PV of $20 = $18.10

 $ 6.90

The price of the call must be $6.90

**Determining how much of the stock to buy**

 Delta = Swing of call / Swing of stock = $10 - $0 / $60 - $40 = 10/20 = ½

 Swing = Maximum value minus minimum value

 The stock could be worth a max of 60 and a min of 40 so the swing is 20

 The call could be worth a max of 10 (if the stock is worth 60) and a min of 0 (if

 the stock is worth 20)

 Since the swing of the call is ½ the swing of the stock, the delta is ½ and that’s

 how much of the stock to buy.

**Determining how much to borrow**

 Once we know how much stock to buy, we can determine how much we need to borrow to equate the payoffs.

 Buying ½ share means we’ll get either $30 or $20 at expiration (from the stock). Compare this to the $10 or $0 we’ll get from the call. It is $20 too much, so we need to set up a situation where we have to pay out $20 on the expiration date. The PV of $20 is $18.18 when interest rates are 10% and it is one year till the expiration date.

Note that we don’t need to know the probabilities of the stock going to $60 or $40. The probabilities are already built in to the price of the stock ($50).

Since we don’t need to know the probabilities, we can price options as if investors are risk-neutral, even though they are actually risk-averse.

Value of Call = (Stock Price) (Delta) – Amount Borrowed

We can then value a Put using Put-Call Parity:

Value of Put = Value of Call + PV Exercise Price – Stock Price